wxMaxima pour l'Analyse Volume I

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Exercices du Chapitre 0 et leur corrigé avec wxMaxima

Présentation

Le manuel wxMaxima for Calculus Volume I, dont l'auteur est Zachary Hannan, propose l'étude de nombreux domaines de l'analyse à l'aide du logiciel Maxima et de son interface graphique wxMaxima. A l'occasion de la traduction française de ce manuel, la correction des exercices de chaque chapitre est proposée. Ce document reprend les exercices du manuel et leur correction à l'aide de wxMaxima.

La dernière version de ce document, ainsi que les fichiers source pour wxMaxima, sont téléchargeables sur le site https://maxima-french-doc.fr/, à la rubrique Documentation.

Enoncé des exercices

1. Définir les variables $A = \sqrt{3}$ et $B = 5$, puis trouver une approximation numérique pour $A + B$ et A/B.
2. Définir les expressions $A = x^2$ and $B = e^x$. Dans l'expression de A, remplacer x par B, puis évaluer l'expression obtenue pour $x = 0.1$ et en donner une approximation décimale.
3. Utiliser `trigexpand` pour trouver une formule donnant $\sin(x + y)$ en fonction de $\sin x$, $\sin y$, $\cos x$ et $\cos y$. Utiliser ce résultat pour calculer $\sin \frac{5\pi}{12}$ en se servant du fait que $\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$. Re-calculer $\sin \frac{5\pi}{12}$ directement et utiliser `float` afin de vérifier votre réponse.
4. Additionner et simplifier : $\frac{2x^2-x-6}{x^2-9} + \frac{x^3-x^2-4x+4}{x^2+5x+6}$. Exprimer la réponse sous forme factorisée.
5. Résoudre l'équation $ax^2 + bx + c = 0$ en x.
6. Représenter graphiquement $f(x) = \sin(\ln x) - 0.2$ sur $[10, 30]$ en incluant l'axe des abscisses. Utiliser `find_root` pour trouver une valeur approchée de la solution de $\sin(\ln x) = 0.2$ sur cet intervalle. Vérifier la réponse en calculant $\sin(\ln x)$ à la valeur de x trouvée.
7. Définir $f(x) = x + 2$ et $g(x) = x^2$. Trouver les intersections de ces deux courbes, puis représenter graphiquement les deux fonctions en dessinant les points d'intersection comme des disques. Nommer chaque point d'intersection en utilisant ses coordonnées décimales arrondies à la deuxième décimale.
8. Utiliser `makelist` et `for-do` pour générer les trente premiers termes de la suite $1, \frac{1}{2}, \frac{1}{4}, \ldots$.
9. La formule de récurrence d'une suite est donnée par $f_n = 2f_{n-1}$ avec comme premier terme $f_0 = 3$. Utilisez une boucle 'do' pour générer récursivement les 10 premiers termes de cette suite (comme dans l'exemple 0.5.4). Une fois la suite écrite, essayez de deviner une formule explicite pour f_n. Une fois cette formule trouvée, utilisez `makelist` pour générer la même suite.
10. Utilisez `makelist` pour tracer 40 cercles centrés à l'origine avec des rayons $0.1, 0.2, 0.3, \ldots$ dans un seul graphique. Ce problème est délicat, car cette liste doit produire des éléments que `wxdraw2d` sait interpréter : `implicit` et ses arguments appropriés doivent être inclus dans chaque élément de la liste !
11. Toute parabole peut s'écrire sous la forme $f(x) = ax^2 + bx + c$. Les paramètres a, b, c définissent complètement la parabole. Si l'on vous donne trois points appartenant à une parabole inconnue, ils génèrent un système de trois équations en a, b, c. La commande `solve` permet de résoudre rapidement ce système. Écrivez une solution similaire à l'exemple 0.6.1 pour définir trois points quelconques et tracer à la fois ces points et la parabole qui passe par ces trois points. La syntaxe correcte pour résoudre un système d'équations est : `solve([eqn1,eqn2,eqn3],[var1,var2,var3])`. Appliquer ce programme aux points $(-6.8, -5.5)$, $(0.1, 6.7)$ et $(3.2, -0.9)$.

Corrigé des exercices

wxMaxima pour l'analyse, volume I
Correction des exercices du chapitre 0
Fichier : correction-chapitre0.wxmx

```
load(draw);
```

/usr/share/maxima/5.45.1/share/draw/draw.lisp

1 Exercice1

```
A:sqrt(3);B:5;
```

$\sqrt{3}$

5

```
float(A+B);
```

6.732050807568877

```
float(A/B);
```

0.3464101615137754

2 Exercice 2

```
A:x^2;B:%e^x;
```

x^2

$\%e^x$

```
subst(B, x, A);
```

$\%e^{2x}$

```
ev(%,x=0.1);
```

1.221402758160169

3 Exercice 3

```
trigexpand(sin(x+y));
```

$\cos(x)\sin(y)+\sin(x)\cos(y)$

```
cos(%pi/6);sin(%pi/6);cos(%pi/4);sin(%pi/4);
```

$\frac{\sqrt{3}}{2}$

$\frac{1}{2}$

$\frac{1}{\sqrt{2}}$

$\frac{1}{\sqrt{2}}$

```
result:cos(%pi/6)·sin(%pi/4)+cos(%pi/4)·sin(%pi/6);
```

$\frac{\sqrt{3}}{2^{3/2}}+\frac{1}{2^{3/2}}$

```
factor(result);
```

$\frac{\sqrt{3}+1}{2^{3/2}}$

```
sin(5·%pi/12);
```

$\sin\left(\frac{5\pi}{12}\right)$

```
float(sin(5·%pi/12));
```

0.9659258262890683

```
float(result);
```

0.965925826289068

4 Exercice 4

```
(2·x^2−x−6)/(x^2−9)+(x^3−x^2−4·x+4)/(x^2+5·x+6);
```

$$\frac{x^3-x^2-4x+4}{x^2+5x+6}+\frac{2x^2-x-6}{x^2-9}$$

```
factor(%);
```

$$\frac{(x-2)\left(x^2-2x+6\right)}{(x-3)(x+3)}$$

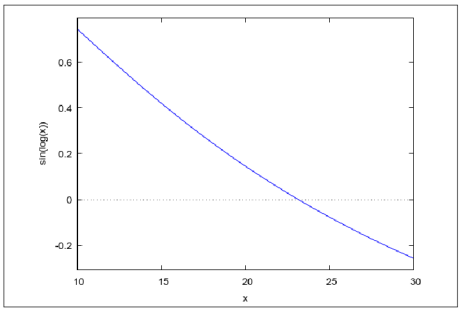
5 Exercice 5

```
solve(a·x^2+b·x+c,x);
```

$$\left[x=-\frac{\sqrt{b^2-4ac}+b}{2a},x=\frac{\sqrt{b^2-4ac}-b}{2a}\right]$$

6 Exercice 6

```
wxplot2d(sin(log(x)),[x,10,30])$
```




```
sol:find_root(sin(log(x)), x, 10, 30);
```

23.14069263277927

```
sin(log(sol));
```

$1.224646799147353\ 10^{-16}$

7 Exercice 7

```
f(x):=x+2;g(x):=x^2;
```

$f(x):=x+2$

$g(x):=x^2$

```
solve(f(x)=g(x),x);
```

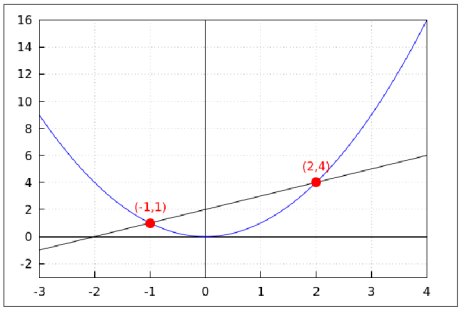
$[x=2,x=-1]$

```
f(2);f(−1);
```

4

1

```
wxdraw2d(
   color=black,
   explicit(0,x,−3,4),
   parametric(0,t, t, −3, 16),
   explicit(f(x),x,−3,4),
   color=blue,
    explicit(g(x),x,−3,4),
   point_type=filled_circle,
   point_size=2,
   color=red,
  points([[−1,f(−1)], [2,f(2)]]),
  grid=true,
   label(["(2,4)", 2, f(2)+1.2]),
    label(["(-1,1)", −1, f(−1)+1.2])
   );
```



8 Exercice 8

```
makelist(1/(2·k),k,1,30);
```

$$[\frac{1}{2},\frac{1}{4},\frac{1}{6},\frac{1}{8},\frac{1}{10},\frac{1}{12},\frac{1}{14},\frac{1}{16},\frac{1}{18},\frac{1}{20},\frac{1}{22},\frac{1}{24},\frac{1}{26},\frac{1}{28},\frac{1}{30},\frac{1}{32},\frac{1}{34},\frac{1}{36},\frac{1}{38},\frac{1}{40},\frac{1}{42},\frac{1}{44},\frac{1}{46},\frac{1}{48},\frac{1}{50},\frac{1}{52},\frac{1}{54},\frac{1}{56},\frac{1}{58},\frac{1}{60}]$$

```
for n:1 thru 30 do
(x:1/(2·n),
print(x));
```

$$\frac{1}{2}$$
$$\frac{1}{4}$$
$$\frac{1}{6}$$
$$\frac{1}{8}$$
$$\frac{1}{10}$$
$$\frac{1}{12}$$
$$\frac{1}{14}$$
$$\frac{1}{16}$$
$$\frac{1}{18}$$
$$\frac{1}{20}$$
$$\frac{1}{22}$$
$$\frac{1}{24}$$
$$\frac{1}{26}$$
$$\frac{1}{28}$$
$$\frac{1}{30}$$
$$\frac{1}{32}$$
$$\frac{1}{34}$$
$$\frac{1}{36}$$
$$\frac{1}{38}$$
$$\frac{1}{40}$$
$$\frac{1}{42}$$
$$\frac{1}{44}$$
$$\frac{1}{46}$$
$$\frac{1}{48}$$
$$\frac{1}{50}$$
$$\frac{1}{52}$$
$$\frac{1}{54}$$
$$\frac{1}{56}$$
$$\frac{1}{58}$$
$$\frac{1}{60}$$

done

9 Exercice 9

```
f_0:3$
print("f_0 =",f_0)$
for n:1 thru 9 do
(X:2·f_0,
   print("f_",n,"=",X),
   f_0:X
)$
```

$f_0 = 3$
$f_1 = 6$
$f_2 = 12$
$f_3 = 24$
$f_4 = 48$
$f_5 = 96$
$f_6 = 192$
$f_7 = 384$
$f_8 = 768$
$f_9 = 1536$

:

```
u(n):=3·2^n;
```

$$u(n):=3\,2^{n}$$

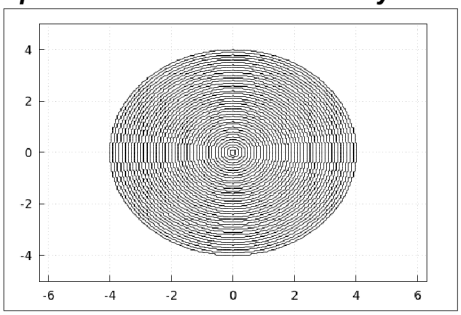
```
makelist(u(n),n,0,9);
```

$$[3,6,12,24,48,96,192,384,768,1536]$$

10 Exercice 10

```
wxdraw2d(
      grid=true,
   color=black,
apply(append, makelist([implicit(x^2 + y^2 = r^2, x, −6.3, 6.3, y, −5, 5)], r, 0.1, 4, 0.1))
)$
```

rat: replaced −0.01 by −1/100 = −0.01
rat: replaced −0.04000000000000001 by −1/25 = −0.04
rat: replaced −0.09000000000000003 by −9/100 = −0.09
rat: replaced −0.16 by −4/25 = −0.16
rat: replaced −0.25 by −1/4 = −0.25
rat: replaced −0.36 by −9/25 = −0.36
rat: replaced −0.4899999999999999 by −49/100 = −0.49
rat: replaced −0.6399999999999999 by −16/25 = −0.64
rat: replaced −0.8099999999999998 by −81/100 = −0.81
rat: replaced −0.9999999999999998 by −1/1 = −1.0
rat: replaced −1.209999999999999 by −121/100 = −1.21
rat: replaced −1.44 by −36/25 = −1.44
rat: replaced −1.69 by −169/100 = −1.69
rat: replaced −1.96 by −49/25 = −1.96
rat: replaced −2.250000000000001 by −9/4 = −2.25
rat: replaced −2.560000000000001 by −64/25 = −2.56
rat: replaced −2.890000000000001 by −289/100 = −2.89
rat: replaced −3.240000000000001 by −81/25 = −3.24
rat: replaced −3.610000000000002 by −361/100 = −3.61
rat: replaced −4.000000000000002 by −4/1 = −4.0
rat: replaced −4.410000000000002 by −441/100 = −4.41
rat: replaced −4.840000000000002 by −121/25 = −4.84
rat: replaced −5.290000000000004 by −529/100 = −5.29
rat: replaced −5.760000000000004 by −144/25 = −5.76
rat: replaced −6.250000000000004 by −25/4 = −6.25
rat: replaced −6.760000000000005 by −169/25 = −6.76
rat: replaced −7.290000000000005 by −729/100 = −7.29
rat: replaced −7.840000000000006 by −196/25 = −7.84
rat: replaced −8.410000000000008 by −841/100 = −8.41
rat: replaced −9.000000000000009 by −9/1 = −9.0
rat: replaced −9.610000000000009 by −961/100 = −9.61
rat: replaced −10.24 by −256/25 = −10.24
rat: replaced −10.89000000000001 by −1089/100 = −10.89
rat: replaced −11.56000000000001 by −289/25 = −11.56
rat: replaced −12.25000000000001 by −49/4 = −12.25
rat: replaced −12.96000000000001 by −324/25 = −12.96
rat: replaced −13.69000000000001 by −1369/100 = −13.69
rat: replaced −14.44000000000001 by −361/25 = −14.44
rat: replaced −15.21000000000001 by −1521/100 = −15.21
rat: replaced −16.00000000000001 by −16/1 = −16.0



```
cercles:makelist([implicit(x^2 + y^2 = r^2, x, −4, 4, y, −4, 4)], r, 0.1, 4, 0.1);
```

$$[[\mathrm{implicit}(y^2+x^2=0.01,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=0.04000000000000001,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=0.09000000000000003,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=0.16,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=0.25,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=0.36,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=0.4899999999999999,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=0.6399999999999999,x,-4,4,y,-4,4)]$$
$$,[\mathrm{implicit}(y^2+x^2=0.8099999999999998,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=0.9999999999999998,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=1.209999999999999,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=1.44,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=1.69,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=1.96,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=2.250000000000001,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=2.560000000000001,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=2.890000000000001,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=3.240000000000001,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=3.610000000000002,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=4.000000000000002,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=4.410000000000002,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=4.840000000000002,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=5.290000000000004,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=5.760000000000004,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=6.250000000000004,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=6.760000000000005,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=7.290000000000005,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=7.840000000000006,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=8.410000000000008,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=9.000000000000009,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=9.610000000000009,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=10.24,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=10.89000000000001,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=11.56000000000001,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=12.25000000000001,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=12.96000000000001,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=13.69000000000001,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=14.44000000000001,x,-4,4,y,-4,4)],[\mathrm{implicit}(y^2+x^2=15.21000000000001,x,-4,4,y,-4,4)],$$
$$[\mathrm{implicit}(y^2+x^2=16.00000000000001,x,-4,4,y,-4,4)]]$$

```
apply(append,cercles);
```

$$[\mathrm{implicit}(y^2+x^2=0.01,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=0.04000000000000001,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=0.09000000000000003,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=0.16,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=0.25,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=0.36,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=0.4899999999999999,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=0.6399999999999999,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=0.8099999999999998,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=0.9999999999999998,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=1.209999999999999,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=1.44,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=1.69,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=1.96,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=2.250000000000001,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=2.560000000000001,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=2.890000000000001,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=3.240000000000001,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=3.610000000000002,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=4.000000000000002,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=4.410000000000002,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=4.840000000000002,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=5.290000000000004,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=5.760000000000004,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=6.250000000000004,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=6.760000000000005,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=7.290000000000005,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=7.840000000000006,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=8.410000000000008,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=9.000000000000009,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=9.610000000000009,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=10.24,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=10.89000000000001,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=11.56000000000001,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=12.25000000000001,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=12.96000000000001,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=13.69000000000001,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=14.44000000000001,x,-4,4,y,-4,4),$$
$$\mathrm{implicit}(y^2+x^2=15.21000000000001,x,-4,4,y,-4,4),\mathrm{implicit}(y^2+x^2=16.00000000000001,x,-4,4,y,-4,4)]$$

11 Exercice 11

```
point(u,v):=[u,v];
```

$$\mathrm{point}(u,v):=[u,v]$$

```
f(x):=a·x^2+b·x+c;
```

$$\mathrm{f}(x):=a\,x^2+b\,x+c$$

```
coefficients:solve([f(xA)=yA,f(xB)=yB,f(xC)=yC],[a,b,c]);
```

$$[[a=\frac{-xB\,yC+2\,yC+(xC-2)\,yB+(xB-xC)\,yA}{2\,xC^2+xB\left(4-xC^2\right)-4\,xC+xB^2\,(xC-2)},b=-\frac{-xB^2\,yC+4\,yC+\left(xC^2-4\right)yB+\left(xB^2-xC^2\right)yA}{2\,xC^2+xB\left(4-xC^2\right)-4\,xC+xB^2\,(xC-2)},c=$$
$$\frac{-2\,xB^2\,yC+4\,xB\,yC+\left(2\,xC^2-4\,xC\right)yB+\left(xB^2\,xC-xB\,xC^2\right)yA}{2\,xC^2+xB\left(4-xC^2\right)-4\,xC+xB^2\,(xC-2)}]]$$

```
xA:-6.8;yA:-5.5;xB:0.1;yB:6.7;xC:3.2;yC:-0.9;
```

$$-6.8$$
$$-5.5$$
$$0.1$$
$$6.7$$
$$3.2$$
$$-0.9$$

```
coefficients:solve([f(xA)=yA,f(xB)=yB,f(xC)=yC],[a,b,c]);
```

rat: replaced 5.5 by 11/2 = 5.5
rat: replaced 46.23999999999999 by 1156/25 = 46.24
rat: replaced −6.8 by −34/5 = −6.8
rat: replaced −6.7 by −67/10 = −6.7
rat: replaced 0.01 by 1/100 = 0.01
rat: replaced 0.1 by 1/10 = 0.1
rat: replaced 0.9 by 9/10 = 0.9
rat: replaced 10.24 by 256/25 = 10.24
rat: replaced 3.2 by 16/5 = 3.2

$$\left[\left[a=-\frac{4513}{10695},b=-\frac{37757}{35650},c=\frac{3641717}{534750}\right]\right]$$

```
a:rhs(apply(flatten,coefficients)[1]);
```

$$-\frac{4513}{10695}$$

```
b:rhs(apply(flatten,coefficients)[2]);
```

$$-\frac{37757}{35650}$$

```
c:rhs(apply(flatten,coefficients)[3]);
```

$$\frac{3641717}{534750}$$

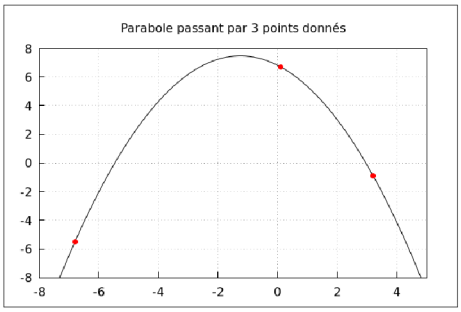
```
f(x);
```

$$-\frac{4513\,x^2}{10695}-\frac{37757\,x}{35650}+\frac{3641717}{534750}$$

```
load(draw);
```

/usr/share/maxima/5.45.1/share/draw/draw.lisp

```
wxdraw2d(
   yrange=[−8,8],
    title="Parabole passant par 3 points donnés",
grid=true,
xaxis=true,
yaxis=true,
color=black,
explicit(f(x),x,−8,5),
   color=red,
point_type=7,
points([point(xA,yA),point(xB,yB),point(xC,yC)])
);
```




```
f(x);
```

$$-\frac{4513\,x^2}{10695}-\frac{37757\,x}{35650}+\frac{3641717}{534750}$$

```
point(xA,yA);point(xB,yB);point(xC,yC);
```

$[-6.8,-5.5]$

$[0.1,6.7]$

$[3.2,-0.9]$